THE chess KNIGHT and the TURK' s head knot

Certainly you have seen a chess board in the past and may be know that among the playing pieces is THE KNIGHT with a particular displacement L shape move.
A knight moves following the diagonal of a 3 x 2 rectangle : jump 2 squares in line with colum or a row then a side jump either on the left or on the right. Knight starts from one 'corner' and end on the diagonally opposite corner.
Which means that starting from a white square it ends on a black square and vice versa (this is very important to remember when you have the knight of you partner in the vicinity : only the squares with the same colour as the square on which is the knight are immune from it.)

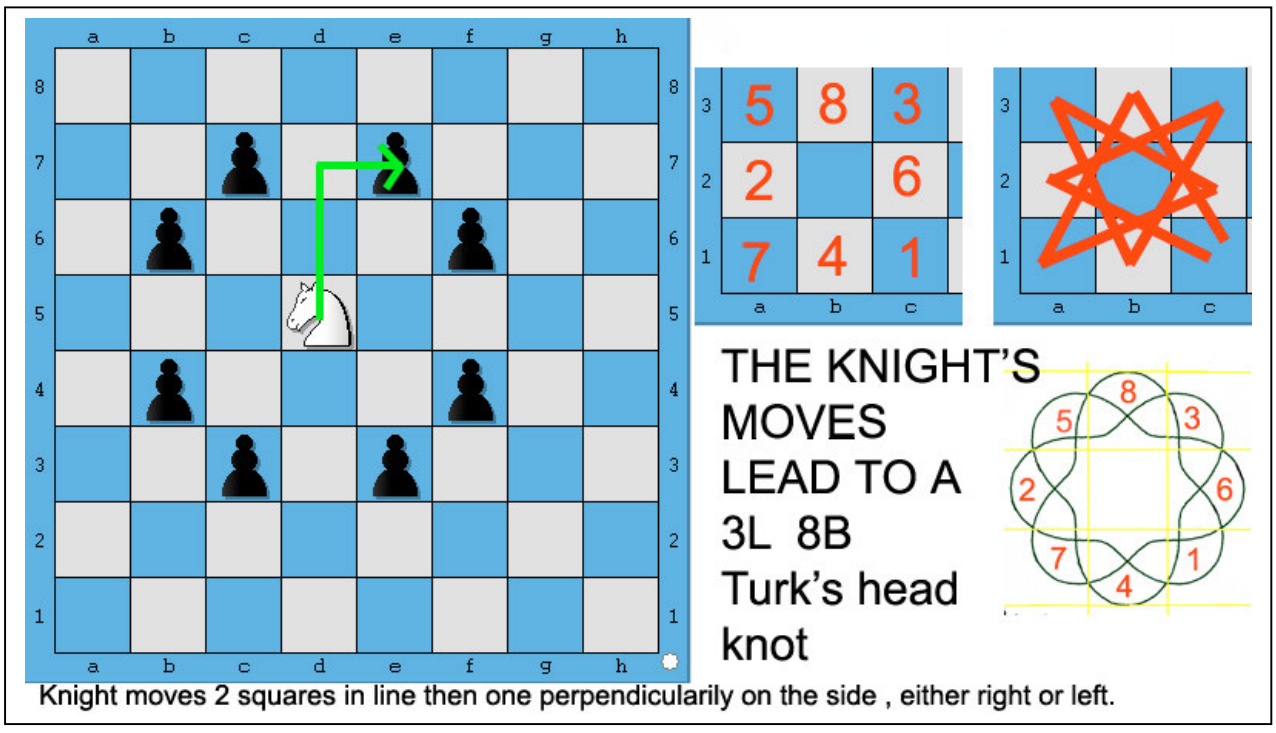
There is a problem that have much preoccupied mathematics : the jumps of a knight (the knight's walk) , or how to cover all 64 squares of a chess board without ever passing twice on the same square or how to make a circuit that is ending in the starting square (closed walk).

One of the first, after the 9th century Arab mathematicians and Euler, to have preoccupied himself with that is Alexandre-Théophile VANDERMONDE (1735-1796) a French precursor polymath. Vandermonde even mentionned knots in a "Mémoire" written for l'Académie Royale des Sciences (Paris - France) with the title of :

"Remarques sur les problèmes de situation", (that is "Remarks about placement problems" or situation problems) pages 566-574. (Vandermonde's matrix is still in everyday use in our days.)

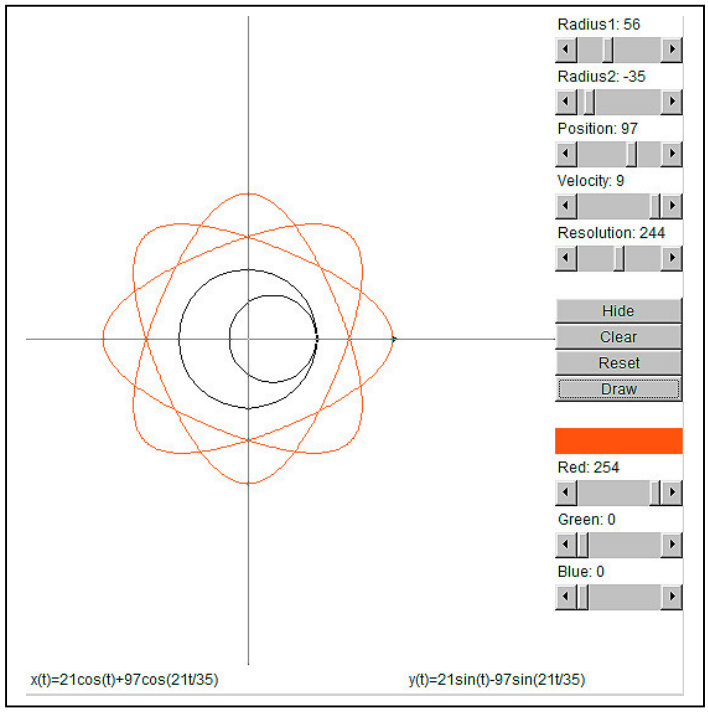
Vandermonde studied topological problem much before the German Johann Benedict LISTING (1808-1882) in mid-19th coined the word TOPOLOGY (he was under the shadow of the Great GAUSS who just supervised LISTING's work and never worked really on knots)
It is much later that the UK mathematicians preoccupied themselves with topology and knots.
By the way Listing discovered before Moebius what is now known as a Moebius's strip.

On a 3 x 3 area of a chess board a knight can jump on 8 of the 9 squares.
Its walk can be used to drawn a 3L 8B THK template.
Just see for your self.
Were you apprised of that before I told you my secret ?



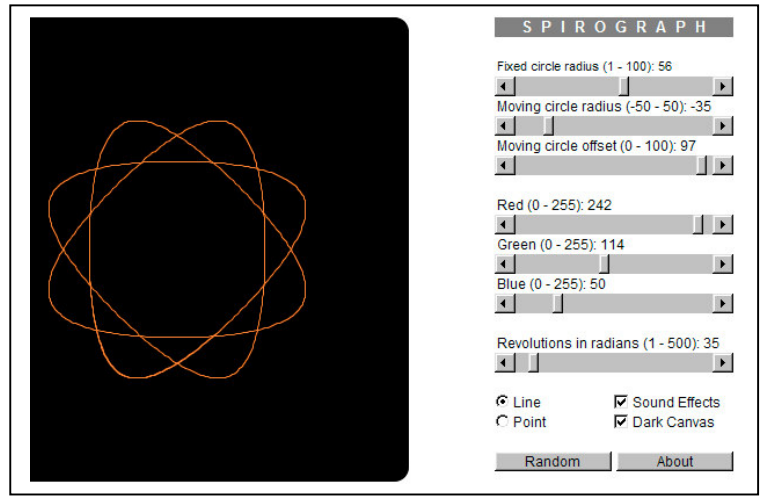
Another 'secret' : the SPIROGRAPH : very useful to draw THK templates.

Spirograph draw TROCHOID curves and THK are TROCHOID. (Sorry I stay with what 17th French mathematician called La Roulette. TROCHOID is just the generalization of CYCLOID that is why I prefer the more extensive word)

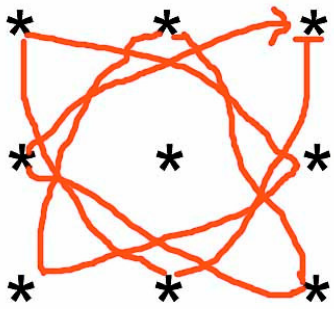


It is not really difficult to infer a rule for getting without fail the THK you want by judiciously choisijng Radius 1, Radius 2 and Position.
Try for a rather high Resolution and let Velocity by easy to follow.

A full issue on the mathematics of THK will come one day.



3L 8B THK as an EULERIAN GRAPH
= a closed walk, a circuit passing once on EACH vertex.



That is so because each vertex is EVEN : an even number of 'arc', edges exist at each of them.

All vertices are EVEN : each of them can be the beginning and ending of a circuit passing by each vertex once.
All vertices SAVE 2 are EVEN : a walk exist passing by each vertex begining at one ODD vertex and ending at the other.

THK and other MATS may be analysed as GRAPH. (if you are interested I will treat the case of MATS but someone wil have to ask for it !).